Int. J. Heat Mass Transfer. Vol. 17, pp. 85–92. Pergamon Press 1974. Printed in Great Britain

VISCOUS DISSIPATION EFFECTS ON UNSTEADY FREE CONVECTIVE FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH VARIABLE SUCTION

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(*Received* 7 *September* 1972 *and in revised form* 20 *April* 1973)

Abstract—A study of a two-dimensional, unsteady flow of an incompressible, viscous dissipative fluid past an infinite plate with variable suction has been presented. Approximate solutions to the coupled non-linear equations governing the flow are derived and expressions for the fluctuating parts of the velocity, transient velocity and temperature, the amplitude and the phase of the skin-friction and the rate of heat transfer are derived. The mean flow is affected by G (Grashof number), E (Eckert number) and P (Prandtl number) and the fluctuating flow is affected by G, E, P, A (suction parameter) and ω (frequency). During the course of discussion, the mean flow and the fluctuating flow are discussed separately.

NOMENCLATURE

A,	suction parameter,
$\lvert B \rvert$,	amplitude of the skin-friction;
C_p,	specific heat at constant pressure;
E,	Eckert number;
f_x,	gravitational force;
G	Grashof number;
k,	thermal conductivity;
M_r, M_i	fluctuating parts of the unsteady velocity;
P	Prandtl number;
q'	rate of heat transfer;
T',	temperature in the boundary layer;
T'_∞,	free stream temperature;
T'_w,	plate temperature;
T_r, T_i,	fluctuating parts of the unsteady temperature;
t',	time;
u', v',	velocity components in the x', y'-directions;
v_0,	suction velocity;
x', y',	coordinate axes along and perpendicular to the plate;
β,	coefficient of volume expansion;
ρ',	density;
μ,	viscosity;
ν,	kinematic viscosity;
ω',	frequency;
ε,	amplitude of the unsteady part;
τ',	skin-friction;
α,	phase of the skin-friction;
α_1,	phase of the rate of heat transfer.

1. INTRODUCTION

INTEREST in free convective flow of a viscous, incompressible fluid past a semi-infinite or infinite flat plates is taken by many researchers. Usually, it is assumed that in such flows the viscous dissipative heat is negligible. But Gebhart [1] has shown that the viscous dissipative heat is important when the natural convection flow field is of extreme size or the flow is at extremely low temperature or in high gravity field. Gebhart and Mollendorf [2] have again studied the effects of viscous dissipative heat on free convective flow past semi-infinite plates. Many papers, published on this topic, deal with steady flow, but unsteady free convective flow past bodies of different shapes has received little attention. With suction at vertical plate, the unsteady free convective flow problems, without viscous dissipative heat, have been studied by Nanda and Sharma [3] and Pop [4]. In [3], the suction velocity was assumed to be proportional to $t^{-\frac{1}{2}}$ whereas in [4], time-dependent oscillatory type of suction velocity was assumed. The effects of magnetic field on the flow studied in [4] have been studied independently by Pop [5] and Soundalgekar [6]. In all these unsteady problems, the plate temperature was assumed to be oscillating about a constant non-zero mean.

With viscous dissipative heat included in the energy equation, the unsteady free convective flow past an infinite porous plate with constants suc tion and the plate temperature oscillating about the constant non-zero mean, was analysed recently by Soundalgekar

[7]. The approximate solutions to the problem governed by coupled non-linear equations were derived by the method suggested by Lighthill [8]. It is now the object of the present paper to study the effects of the variable suction velocity on the unsteady free convective flow of a viscous, dissipative, incompressible fluid past an infinite plate when the plate temperature oscillates with the same frequency as that of the variable suction velocity. Following Lighthill's method, approximate solutions to the coupled non-linear equations are derived in Section 2 and expressions for the mean velocity, fluctuating part of the velocity, transient velocity, transient temperature profiles, the amplitude and the phase of the skin-friction and the rate of heat-transfer are derived and are shown graphically. This is followed by a discussion wherein the results for variable suction velocity are compared with those of constant suction velocity. The mean velocity, affected by the Grashof number G, the Prandtl number P and the Eckert number E, was not discussed in [7]. In the present study, these effects are also discussed.

2. MATHEMATICAL ANALYSIS

Here, a two-dimensional unsteady flow of an incompressible, viscous, dissipative fluid past an infinite porous plate is assumed. The x'-axis is taken along the plate in the vertical direction and the y'-axis is chosen normal to the plate. The fluid is assumed to have constant properties except that the influence of the density variations with temperature is considered only in the body force term. It is also assumed that the variations in density in the body-force term does not affect the other terms of the momentum and energy equations. The variation of expansion coefficient with temperature is assumed to be negligible. Then the physical variables are functions of y' and t' only. Hence following equations govern the flow:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = f_x\beta(T' - T'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} \quad (1)$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'} \quad (2)$$

$$\frac{\partial v'}{\partial y'} = 0 \quad (3)$$

$$\rho' C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = K\frac{\partial^2 T'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2. \quad (4)$$

The last term in equation (4) represents the heat due to viscous dissipation. The present analysis describes the flow of fluids rather than gases as the pressure terms are neglected. All the physical variables are defined in notation.

At the plate, the suction velocity is assumed to be oscillating about a non-zero constant mean, equation (3) integrates to

$$v' = -v_0(1 + \varepsilon A\,e^{i\omega' t'}) \quad (5)$$

where v_0 is the mean suction velocity and ε, A are small such that $\varepsilon A \ll 1$. The negative sign in (5) indicates that the suction velocity is directed towards the plate.

Equations (1) and (4), in view of (5) reduce to the following non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + \varepsilon A\,e^{i\omega t})\frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - P(1 + \varepsilon A\,e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 \quad (7)$$

where

$$\begin{aligned} &y = v_0 y'/\nu, \quad t = v_0^2 t'/4\nu \quad \omega = 4\nu\omega'/v_0^2 \\ &u = u'/v_0, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ &G = \frac{\nu f_x \beta(T'_w - T'_\infty)}{v_0^3}, \\ &P = \mu C_p/K, \\ &E = v_0^2/C_p(T'_w - T'_\infty). \end{aligned} \quad (8)$$

The boundary conditions are:

$$\left.\begin{aligned} &u = 0, \quad \theta = \theta_w(t) = 1 + \varepsilon\,e^{i\omega t} \quad \text{at } y = 0 \\ &u \to 0, \quad \theta \to 0 \quad \text{as } y \to \infty \end{aligned}\right\}. \quad (9)$$

Equations (6) and (7) are the coupled non-linear equations governing the flow and are to be solved under the boundary conditions (9). Such a system of equations with $A = 0$ is solved by one of the authors in [7]. Following this procedure, these equations are solved and the solutions for u_0, u_1, θ_0, θ_1 are obtained. To save the space, they are not mentioned here. Now as in [7], the expressions for the transient velocity, the transient temperature, amplitude and the phase of the skin-friction and the rate of heat transfer are derived. They are shown graphically, followed by a discussion.

DISCUSSION

In order to get a physical insight into the problem, numerical calculations are carried out for different values of G, E, P, A and ω. The motion being free convective, E is very small and its values are chosen as 0·01 and 0·02, as these will be more appropriate from the practical point of view. In [7], the values of E were taken as 0·1 and 0·2. Hence the present results are different from those in [7]. In [7], arbitrary values of P were taken. In this study, from realistic point of view, water is considered and hence one of the value of P is chosen as 7, and all the results are shown for water. This is a case for water at 20°C. The behaviour of water at 4°C is quite different from that 20°C as the Prandtl number is also different. So to study the phenomenon of unsteady free convection in water at 4°C, the governing equations should be modified. This study will be presented in a subsequent paper. From the practical point of view, the study of mean flow is important. Hence the discussion is divided into two parts. In part I, the discussion is confined to the mean flow, which was not discussed in [7], and in part II, the discussion is confined to fluctuating flow.

Part I. Mean flow

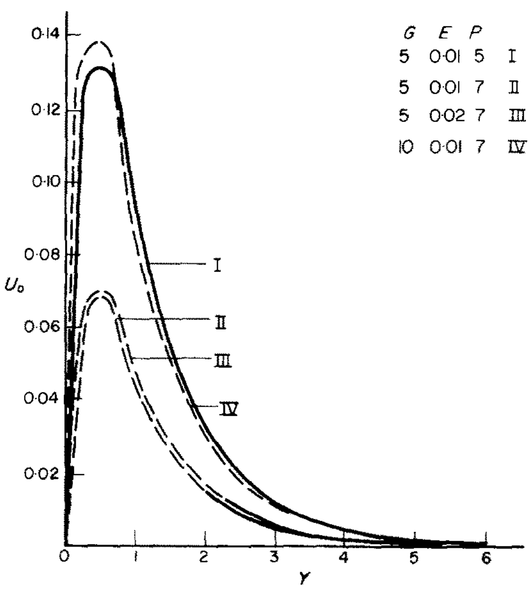

FIG. 1. Mean velocity profile.

The mean velocity profiles for water ($P = 7$) and other fluid are shown in Fig. 1. We observe from this figure that in case of water, when G is changed from 5 to 10, there is a 100 per cent rise in the mean velocity of water. But with greater viscous dissipative heat, there is only 1·4 per cent rise in the mean velocity of water. There is a drop in the mean velocity as the Prandtl number is reduced. In Fig. 2 are shown the mean temperature profiles. When G is doubled, the mean temperature at $y = 0{\cdot}5$ drops by 44 per cent in

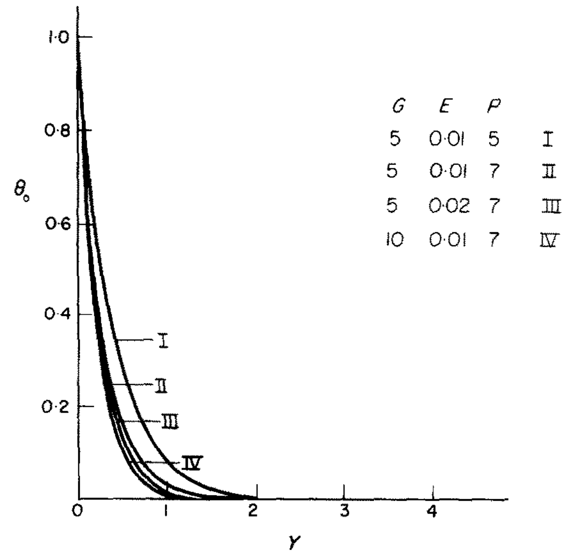

FIG. 2. Mean temperature profile.

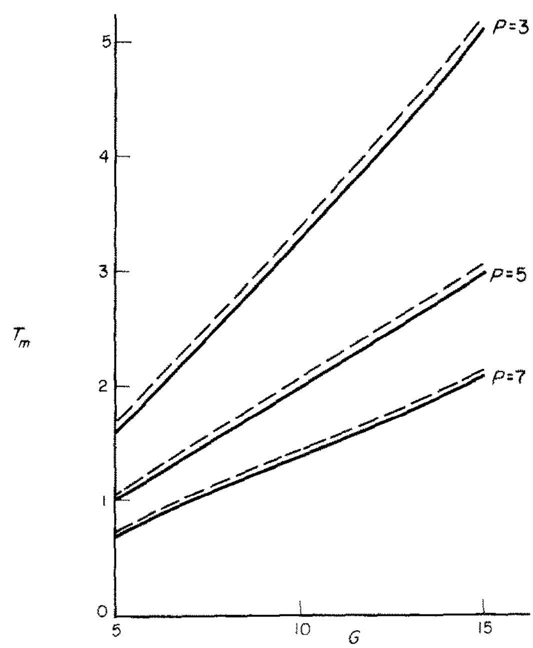

FIG. 3. Mean skin friction $E = 0{\cdot}01$ ——— ; 0·02 – – – .

case of water. With greater viscous dissipative heat, there is also a drop in the mean temperature of water. The mean skin-friction is shown on Fig. 3. An increase in G leads to an increase in the mean skin-friction. For $P = 7, E = 0{\cdot}01$, when G is increased from 5 to 10, there is a rise of 100 per cent in the mean skin-friction. But for $P = 3$, under similar circumstances, there is a 106 per cent rise in the mean skin-friction. This leads us to conclude that the rate of decrease in the mean skin-friction increases with a drop in the Prandtl number. Greater viscous dissipative heat also leads to a rise in the mean skin-friction.

Part II. Fluctuating flow

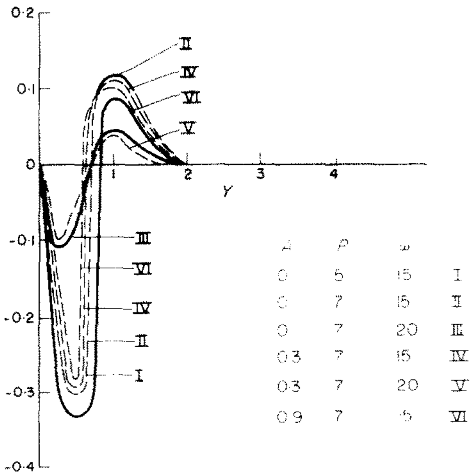

FIG. 4. Fluctuating part of velocity profile, $G = 5$, $E = 0{\cdot}01$.

There is a rise in M_r with increasing ω for all A in case of water. An increase in A leads to an increase in M_r whereas an increase in G or E leads to a decrease in M_r for constant A or ω.

On Figs. 6 and 7, M_i is plotted. There is a rise in M_i owing to the increase in ω. For water, as A is increased from 0·3 to 0·7, there is a 19 per cent fall in M_i, and when G is doubled, there is a 112 per cent fall in M_i. The effect of greater viscous dissipative heat is evident.

On Figs. 8 and 9, the transient velocity profiles are shown. We observe here that for $\omega = 15$, $P = 7$, $G = 5$, $E = 0{\cdot}01$, when A is changed from 0 to 0·3, there is 4·3 per cent rise in the transient velocity. When $P = 7$, $G = 5$, $E = 0{\cdot}01$, and ω is increased from 15 to 20, there is a drop of 50 per cent for $A = 0$ and a drop of 20·8 per cent when $A = 0{\cdot}3$ in the transient velocity. This leads us to conclude that in the presence

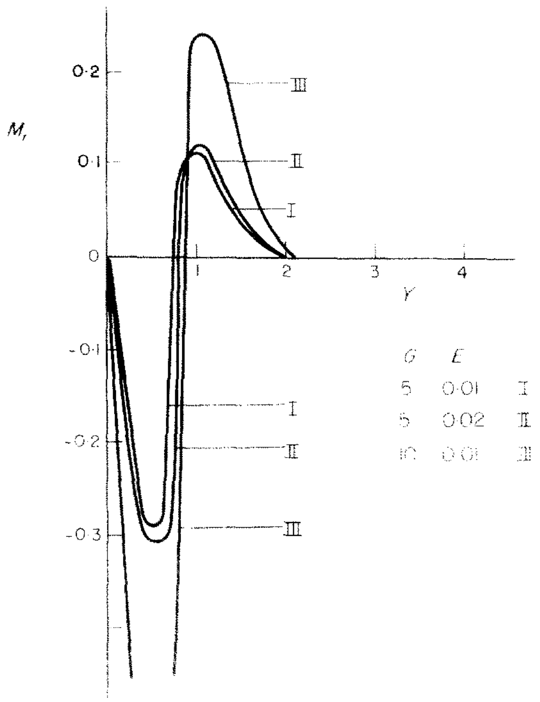

FIG. 5. Fluctuating part of velocity profile, $P = 7$, $\omega = 15$, $A = 0{\cdot}3$.

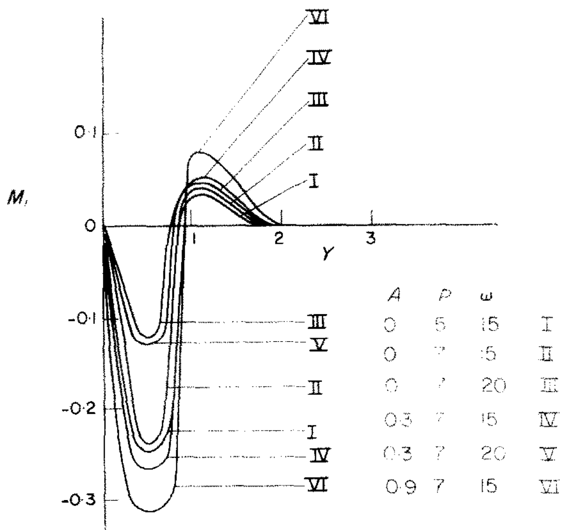

FIG. 6. Fluctuating part of velocity profile. $G = 5$, $E = 0{\cdot}01$.

of the variable suction velocity, the rate of decrease in the transient velocity is more as compared to that in the presence of a constant suction velocity. The

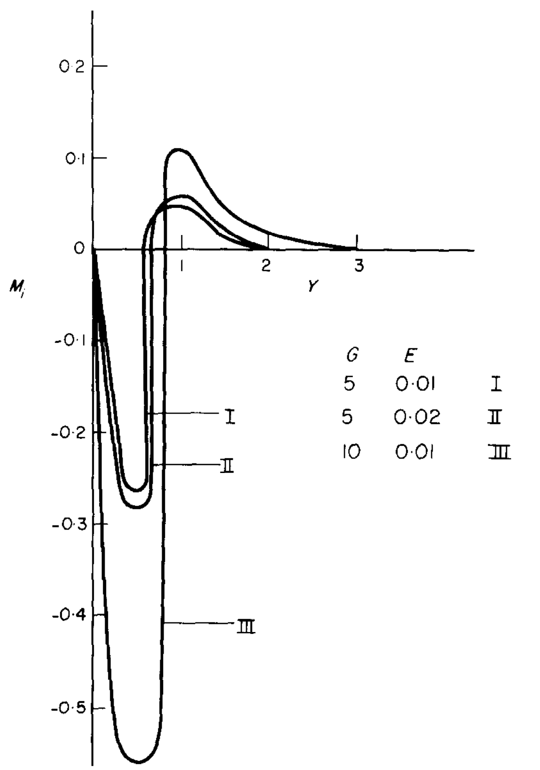

FIG. 7. Fluctuating part of velocity profiles, $p = 7$, $\omega = 15$, $A = 0{\cdot}3$.

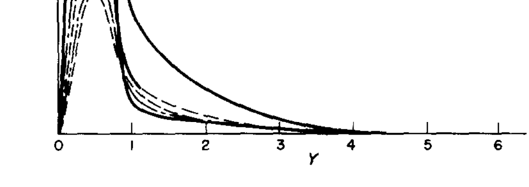

FIG. 8. Transient velocity profile, $G = 5$, $E = 0{\cdot}01$, $\varepsilon = 0{\cdot}2$, $\omega t = \pi/2$.

transient velocity also rises with increasing G. Thus from Fig. 9, we observe that when G is doubled, there is a 127 per cent rise in the transient velocity. Greater viscous dissipative heat also causes a rise in the transient velocity.

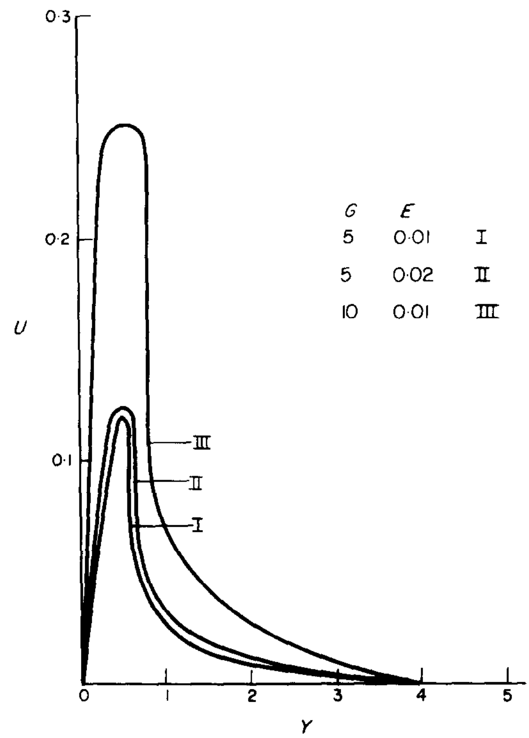

FIG. 9. Transient velocity profile, $A = 0{\cdot}3$, $p = 7$; $\omega = 15$, $\varepsilon = 0{\cdot}2$, $\omega t = \pi/2$.

As a negligible difference is observed between the mean temperature profiles and the transient temperature profiles, the transient temperature profiles are not shown. We conclude from this that the transient temperature is not affected significantly by the frequency of the oscillating temperature of the plate.

The amplitude of the skin-friction is shown on Fig. 10. While the amplitude rises when the suction velocity is a variable i.e. for non-zero values of A. Thus when A is changed from zero to 0·3, there is a rise of 4·1 per cent in the amplitude of the skin-friction for $\omega = 7$. For $A = 0{\cdot}3$, $\omega = 7$, there is a 133 per cent rise in the amplitude due to a rise in G from 5 to 10. It also rises with increasing E.

The phase of the skin-friction is shown on Fig. 11. As it is observed to be negative for all values of ω or A, we conclude that there is always a phase-lag.

The amplitude of the rate of heat transfer is plotted on Fig. 12. With rising P, there is a rise in the amplitude $|Q|$. In case of water, when A is changed from zero to 0·3, there is a rise of 4·3 per cent in $|Q|$ when $\omega = 7$. Also, the rate of rise in $|Q|$ due to an increase in G from 5 to 10 is very large in case of water. With greater viscous dissipative heat, there is also a rise in $|Q|$.

The phase of the rate of heat transfer $\tan \alpha_1$, is shown on Fig. 13. For small ω, it is negative whereas for

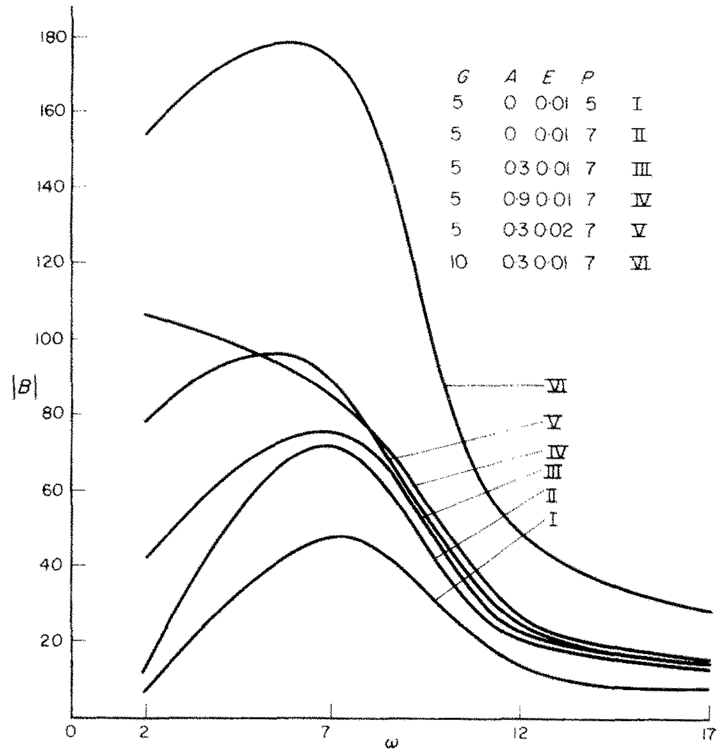

FIG. 10. Amplitude of skin friction.

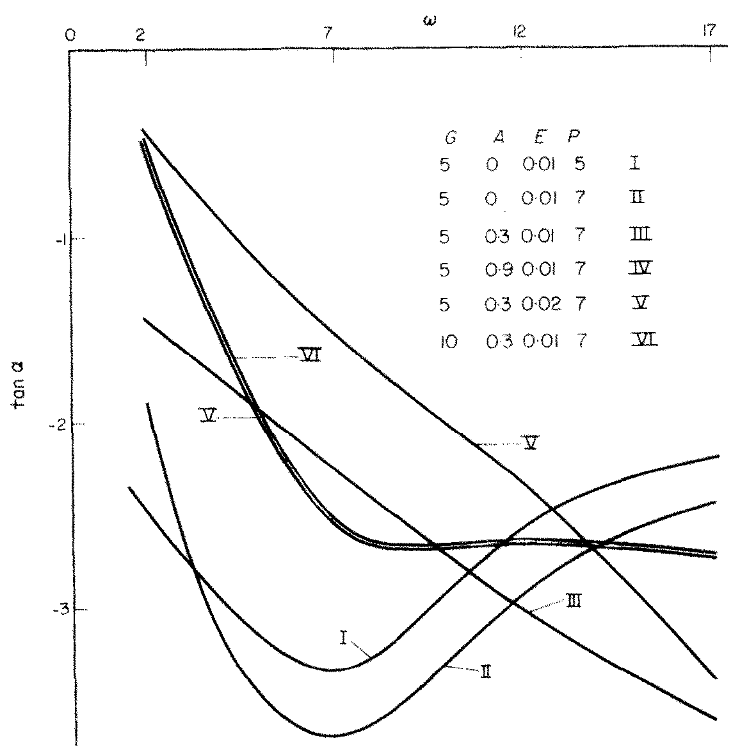

FIG. 11. Phase of skin friction.

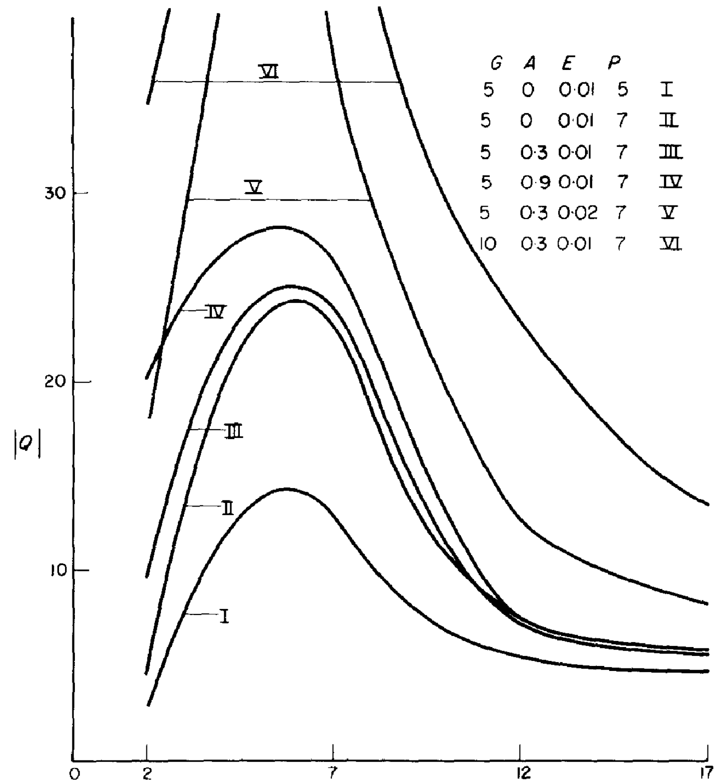

FIG. 12. Amplitude of rate of heat transfer.

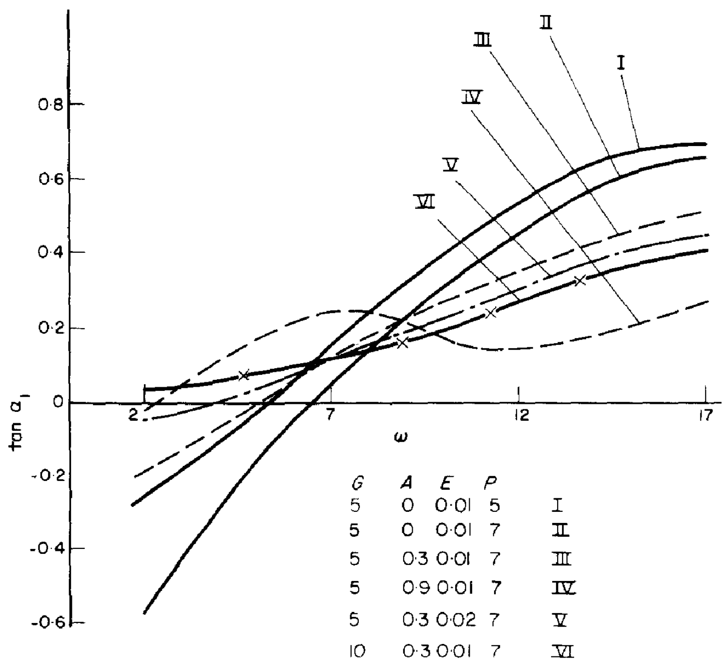

FIG. 13. Phase of rate of heat transfer.

large ω, it is positive. Hence there is a phase-lead at large ω. With increasing A, the phase increases at small ω and decreases at large ω. Same is the effect of G or E.

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EFFET DE LA DISSIPATION VISQUEUSE SUR LA CONVECTION NATURELLE INSTABLE LE LONG D'UNE PLAQUE VERTICALE POREUSE AVEC UNE SUCCION VARIABLE

Résumé—On présente l'étude de l'écoulement instationnaire et bidimensionnel d'un fluide incompressible à dissipation visqueuse le long d'une plaque infinie avec une succion variable. On obtient des solutions approchées des équations non linéaires couplées gouvernant l'écoulement et on présente des expressions pour les parts fluctuantes de la vitesse, la vitesse et la température transitoires, l'amplitude et la phase du frottement pariétal et pour le flux thermique. L'écoulement moyen est affecté par G (nombre de Grashof), E (nombre d'Eckert) et P (nombre de Prandtl) et l'écoulement fluctuant est affecté par G, E, P, A (paramètre de succion) et ω (fréquence). Dans le cours de la discussion, l'écoulement moyen et l'écoulement fluctuant sont considérés séparément.

VISKOSE DISSIPATIONSEFFEKTE BEI NICHTSTATIONÄRER, FREIER KONVEKTION LÄNGS EINER UNENDLICHEN, SENKRECHTEN PORÖSEN PLATTE MIT VARIABLER ABSAUGUNG

Zusammenfassung—Es wurde eine zweidimensionale nichtstionäre Strömung eines inkompressiblen, viskos- dissipativen Fluids längs einer unendlichen Platte mit variabler Absaugung untersucht. Die Näherungs-Lösungen der die Strömung beschreibenden gekoppelten, nichtlaminaren Gleichungen wurden bestimmt sowie Ausdrücke für die fluktierenden Anteile der Geschwindigkeit, des Geschwindigkeitsübergangs und der Temperatur, der Amplitude und der Phase der Oberflächenreibung und des Wärmeübergangs. Die mittlere Strömung wird durch die Grashof-Zahl (*Gr*), die Eckert-Zahl (*Ec*) sowie die Prandtl-Zahl (*Pr*) beeinflusst und die fluktuierende Strömung durch die Kennzahlen *Gr*, *Ec*, *Pr*, sowie den Absaugparameter A und die Frequenz ω. Während des Verlaufs der Dikussion wurde die mittlere und die fluktuierende Strömung getrennt behandelt.

ВЛИЯНИЕ ВЯЗКОЙ ДИССИПАЦИИ НА НЕСТАЦИОНАРНУЮ СВОБОДНУЮ КОНВЕКЦИЮ У БЕСКОНЕЧНОЙ ВЕРТИКАЛЬНОЙ ПОРИСТОЙ ПЛАСТИНЫ ПРИ ПЕРЕМЕННОМ ОТСОСЕ

Аннотация—Рассматривается двумерная задача нестационарного обтекания бесконечной пластины несжимаемой вязкой диссипативной жидкостью при переменном отсосе. Получены приближенные решения системы нелинейных уравнений для течения, а также выражения для осциллирующих компонентов скорости, нестационарных скорости и температуры, поверхностного трения и интенсивности теплообмена. На основной поток оказывают влияние число Грасгофа (G), число Эккерта (E) и число Прандтля (P), а пульсационное течение зависит от G, E, P, параметра отсоса A и частоты ω. Среднее и пульсационное течения рассматриваются в отдельности.